1. Algebra

1 The properties of powers

i) $c^0 = 1$

ii) $c^{-x} = \frac{1}{c^x}$

iii) $c^{\frac{x}{y}} = \sqrt[y]{c^x} = \left(\sqrt[y]{c}\right)^x$

iv) $\left(c^x\right)^y = c^{xy} = \left(c^y\right)^x$

v) $c^x c^y = c^{x+y}$

vi) $\frac{c^x}{c^y} = c^{x-y}$

2 Manipulating expressions.

$a+b=b+a$

$(a+b)+c=a+(b+c)=(a+c)+b$

$ab=ba$

$(ab)c=a(bc)=b(ac)$

3 Some useful algebraic identities.

$a^2-b^2=(a-b)(a+b)$ or, more generally,

$a^n-b^n=\left(a-b\right)\left(a^{n-1}+a^{n-2}b+\ldots+ab^{n-2}+b^{n-1}\right)$

$(a\pm b)^2=a^2\pm 2ab+b^2$

4 Solution of 2nd order polynomial equation of the form $a_2x^2+a_1x+a_0=0$. The "quadratic formula" is

$$x_1, x_2 = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2a_0}}{2a_2}$$

5 Complex numbers are of the form $z=a+bi$ where $i^2=-1$ (or, $i=\sqrt{-1}$). Algebra with complex numbers uses the following identities:

$z_1 \pm z_2 = (a_1 \pm a_2) + (b_1 \pm b_2)i$

$z_1 z_2 = a_1a_2 + (a_1b_2 + a_2b_1)i - b_1b_2$

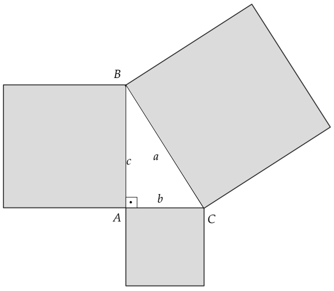
$\frac{z_1}{z_2} = \frac{z_1\overline{z}_2}{z_2\overline{z}_2}$, where $\overline{z}$ is the conjugate of z, such that

$z\overline{z} = (a+bi)(a-bi) = a^2+b^2$.

2. Geometry

1 Pythagora's theorem:

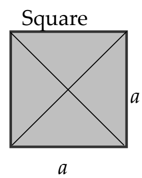
$a^2 = c^2 + b^2$



Distance between two points in 2D space:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

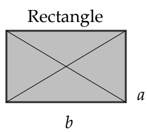
2 Properties of two-dimensional shapes and three-dimensional solids



Diagonals are equal and bisect each other at right angles.

Perimeter: $4a$

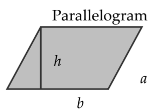
Area: a^2



Diagonals are equal and bisect each other

Perimeter: $2(a+b)$

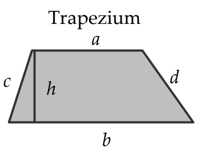
Area: ab



Diagonals bisect each other

Perimeter: $2(a+b)$

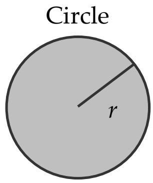
Area: hb



Two sides are parallel

Perimeter: $a+b+c+d$

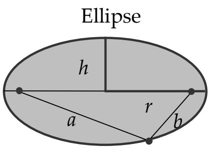
Area: $h\dfrac{a+b}{2}$



All points on the perimeter (circumference) are the same distance (radius) from a point (the centre) internal to the circle.

Perimeter: $2\pi r$

Area: πr^2

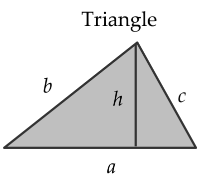


An ellipse is the set of points on a plane whose sum of distances ($a+b$) from two given points is the same.

Perimeter (approximate):

$$\frac{h+r}{2}\left(1+\frac{3\left(\frac{r-h}{r+h}\right)^2}{10+\sqrt{4-3\left(\frac{r-h}{r+h}\right)^2}}\right)$$

Area: πrh

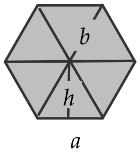


The same area can be calculated using any side of the triangle with its corresponding height.

Perimeter: $a+b+c$

Area: $\dfrac{ha}{2}$

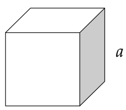
Regular hexagon



All the triangles formed by drawing the three diagonals of the hexagon are equilateral.
Perimeter: $6a$

Area: $3ha = \frac{3\sqrt{3}}{2}a^2$

Cube

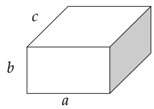


All sides are all squares

Surface area: $6a^2$

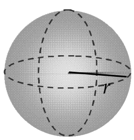
Volume: a^3

Cuboid



All sides are rectangles
Surface area:
$2ab + 2bc + 2ac$
Volume: abc

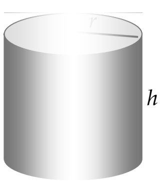
Sphere



Surface area: $4\pi r^2$

Volume: $\frac{4}{3}\pi r^3$

Cylinder



Surface area: $2\pi r(r+h)$

Volume: $\pi r^2 h$

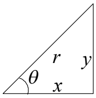
3. Trigonometry

1 Notation. Angles are usually denoted by the Greek letters θ, ϕ, φ

2 Angle conversions between degrees (d) and radians (ϕ): $360 / d = 2\pi / \phi$. Some useful angle conversions are

Degrees	*Radians*	*Degrees*	*Radians*
0	0	135	$3\pi/4$
30	$\pi/6$	150	$5\pi/6$
45	$\pi/4$	180	π
60	$\pi/3$	270	$3\pi/2$
90	$\pi/2$	360	2π
120	$2\pi/3$		

3❧ Definitions of trigonometric functions $\cos(\theta) = x / r$, $\sin(\theta) = y / r$, $\tan(\theta) = y / x$, $\cot(\theta) = x / y$



4❧ Numerical approximation of basic trigonometric functions:

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots \qquad \cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots$$

5❧ Converting polar to Cartesian coordinates: $(x, y) = (r\cos(\theta), r\sin(\theta))$

6❧ Basic trigonometric identities:

$$\tan\theta = \sin\theta / \cos\theta \qquad \cos^2(\theta) + \sin^2(\theta) = 1$$
$$\cos(-\theta) = \cos\theta \qquad \sin(-\theta) = -\sin\theta$$
$$\cos(\theta + n2\pi) = \cos\theta \qquad \sin(\theta + n2\pi) = \sin\theta$$
$$\cos(\pi - \theta) = -\cos\theta \qquad \sin(\pi - \theta) = \sin\theta$$

7❧ Trigonometric identities for sums of angles:

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$
$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$
$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

8❧ Trigonometric identities for double angles:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\tan(\theta \pm \phi) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

9❧ Identities for the product of trigonometric functions:

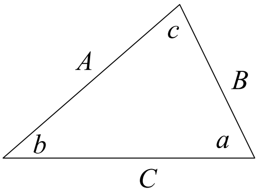
$$\sin\theta\cos\phi = \tfrac{1}{2}\big(\sin(\theta + \phi) + \sin(\theta - \phi)\big)$$
$$\cos\theta\sin\phi = \tfrac{1}{2}\big(\sin(\theta + \phi) - \sin(\theta - \phi)\big)$$
$$\cos\theta\cos\phi = \tfrac{1}{2}\big(\cos(\theta + \phi) + \cos(\theta - \phi)\big)$$
$$\sin\theta\sin\phi = \tfrac{1}{2}\big(\cos(\theta + \phi) - \cos(\theta - \phi)\big)$$

10❧ Identities for the sum of trigonometric functions:

$$\sin\theta+\sin\phi=2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$
$$\sin\theta-\sin\phi=2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$
$$\cos\theta+\cos\phi=2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$
$$\cos\theta-\cos\phi=-2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

11❧ Identities for the angles and sides of an oblique triangle (i.e. one, that does not have a right angle).

The law of sines : $\dfrac{A}{\sin a}=\dfrac{B}{\sin b}=\dfrac{C}{\sin c}$



The law of cosines :
$$A^2=B^2+C^2-2BC\cos a$$
$$B^2=A^2+C^2-2AC\cos b$$
$$C^2=A^2+B^2-2AB\cos c$$

12❧ General transformation of cos into a new periodic function:
$$f(t)=\frac{f_{\max}+f_{\min}}{2}+\frac{f_{\max}-f_{\min}}{2}\cos\left(\frac{2\pi}{T}(t-t_0)\right)$$
where $f_{\min}$ and $f_{\max}$ are the minimum and maximum of the new function (replacing -1 and 1), T is the new period (replacing 2π) and t_0 is the location of the new peak (replacing 0).

13❧ Superposition of several periodic components is described by a trigonometric polynomial
$$f(t)=a+\left(\beta_1\cos\omega t+\beta_2\cos 2\omega t+\cdots\right)+\left(\gamma_1\sin\omega t+\gamma_2\sin 2\omega t+\cdots\right)$$

4. Sequences

1❧ Arithmetic progression. The iterative definition is $a_{n+1}=a_n+m$ and the corresponding general definition is $a_n=a_0+nm$.

2❧ Geometric progression. The iterative definition is $a_{n+1}=ma_n$ and the corresponding general definition is $a_n=m^n a_0$.

3 A taxonomy of difference equations presented using generic examples: c_1, c_2 and c_3 are constants and $c_1(t), c_2(t)$ and $c_3(t)$ are functions of time. The entries describe the right hand side of the difference equation, so a homogeneous, autonomous, linear, affine, equation is $a_{t+1} = c_1 a_t$

	Homogeneous	
	Autonomous	*Non-autonomous*
Linear affine	$c_1 a_t$	$c_1(t) a_t$
Linear	$c_1 a_t + c_0$	$c_1(t) a_t + c_0$
Non-linear affine	$c_2 a_t^2 + c_1 a_t$	$c_2(t) a_t^2 + c_1(t) a_t$
Non-linear	$c_2 a_t^2 + c_1 a_t + c_0$	$c_2(t) a_t^2 + c_1(t) a_t + c_0$

	Non-homogeneous	
	Autonomous	*Non-autonomous*
Linear	$c_1 a_t + c_0(t)$	$c_1(t) a_t + c_0(t)$
Non linear	$c_2 a_t^2 + c_1 a_t + c_0(t)$	$c_2(t) a_t^2 + c_1(t) a_t + c_0(t)$

5. Logarithms

1 Properties of the logarithmic function

i) $\log_c 1 = 0$

ii) $\log_c c = 1$

iii) $c^{\log_c x} = x$

iv) $\log_c(xy) = \log_c x + \log_c y$

v) $\log_c(x/y) = \log_c x - \log_c y$

vi) $\log_c x^k = k \log_c x$

3 Relationship between natural and common logarithms: $\ln x = \ln 10 \log x$. This implies that the two are proportional to each other.

6. Limits

1 Properties of limits.

If $\lim_{x \to a} f(x) = c_1$ and $\lim_{x \to a} g(x) = c_2$, then

i) The limit of the sum equals the sum of the limits

$\lim_{x\to a}\left(f(x)\pm g(x)\right)=c_1\pm c_2$

ii) The limit of the product equals the product of the limits

$\lim_{x\to a}\left(f(x)g(x)\right)=c_1c_2$

iii) The limit of the ratio equals the ratio of the limits, assuming that $c_2\neq 0$

$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\frac{c_1}{c_2}$ given $c_2\neq 0$

iv) The limit of the product between a constant and a function equals the product of the constant and the function's limit

$\lim_{x\to a}\left(cf(x)\right)=cc_1$

2 Limits involving infinity.

i) $\lim_{x\to 0}\frac{1}{x}=\infty$ ii) $\lim_{x\to\infty}\frac{1}{x}=0$

7. Derivatives

1 Definition of the derivative

$$\frac{df}{dx}=\lim_{x\to x_1}\frac{f(x)-f(x_1)}{x-x_1}$$

or, alternatively

$$\frac{df}{dx}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

There are five alternative notations for the derivative of a function:

$$\frac{df}{dx}=\frac{df(x)}{dx}=\frac{d}{dx}f(x)=f'(x)=\dot{f}(x)$$

2 Rules of differentiation

	Function	*Derivative*
1	$f(x)=c$	$f'=0$
2	$f(x)=x^n$	$f'=nx^{n-1}$
3	$f(x)=cg(x)$	$f'=cg'$
4	$f(x)=g(x)+h(x)$	$f'=g'+h'$
5 Product rule	$f(x)=g(x)h(x)$	$f'=g'h+h'g$
6	$f(x)=g(x)^n$	$f'=ng^{n-1}g'$
7 Quotient rule	$f(x)=\frac{g(x)}{h(x)}$	$f'=\frac{g'h-h'g}{h^2}$
8	$f(x)=g(x)^{\frac{n}{m}}$	$f'=\frac{n}{m}g^{\frac{n}{m}-1}g'$

9	$f(x) = \ln x$	$f' = \frac{1}{x}$
10	$f(x) = e^x$	$f' = e^x$
11	$f(x) = \sin x$	$f' = \cos x$
12	$f(x) = \cos x$	$f' = -\sin x$
13	$f(x) = \tan x$	$f' = \sec^2 x$

3 The chain rule

For a function $f(x) = f(g(x))$,

$$\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx}$$

More generally, for $f(x) = f(g_1(g_2(\cdots g_n(x))))$,

$$\frac{df}{dx} = \frac{df}{dg_1}\frac{dg_1}{dg_2}\cdots\frac{dg_n}{dx}$$

4 Taylor expansion

$$f(x) \cong f(x_1) + \sum_{i=1}^{n} \frac{1}{i!} f^{(i)}(x_1)(x - x_1)^i$$

8. Integrals

1 Basic rules of integration

	Function ($F(x)$)	Indefinite integral ($\int F(x)dx$)
1	a	$ax + c$
2	x^n	$\frac{1}{n+1}x^{n+1} + c$
3	$cG(x)$	$c\int G(x)dx$
4	$G(x) + H(x)$	$\int G(x)dx + \int H(x)dx$
5	$G(x)^n \frac{dG}{dx}$	$\frac{G(x)^{n+1}}{n+1} + c$
6	$G(x)^{\frac{n}{m}} \frac{dG}{dx}$	$m\frac{G(x)^{\frac{n}{m}+1}}{n+m} + c$
7	$\frac{1}{x}$	$\ln x + c$
8	e^x	$e^x + c$

9	$\sin x$	$\cos x$
10	$\cos x$	$-\sin x$
11	$\tan x$	$\sec^2 x$

2❧ Manipulating definite integrals

i) Given an interval $[a,b]$ that comprises the two subintervals $[a,c]$ and $[c,b]$ then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

9. Matrices

1❧ Properties of matrix arithmetic. For any three matrices $\mathbf{A},\mathbf{B},\mathbf{C}$ and scalars a,b, the following are true

1 $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$

2 $(\mathbf{A}+\mathbf{B})+\mathbf{C}=\mathbf{A}+(\mathbf{B}+\mathbf{C})$

3 $a(\mathbf{A}+\mathbf{B})=a\mathbf{A}+a\mathbf{B}$

4 $(a+b)\mathbf{A}=a\mathbf{A}+b\mathbf{A}$

5 $\mathbf{A}(\mathbf{BC})=(\mathbf{AB})\mathbf{C}$

6 $\mathbf{IA}=\mathbf{A},\ \mathbf{AI}=\mathbf{A}$

7 $\mathbf{A}(\mathbf{B}+\mathbf{C})=\mathbf{AB}+\mathbf{AC}$

2❧ Properties of the transpose

1 $(\mathbf{A}^T)^T=\mathbf{A}$

2 $(\mathbf{A}+\mathbf{B})^T=\mathbf{A}^T+\mathbf{B}^T$

3 $(\mathbf{AB})^T=\mathbf{B}^T\mathbf{A}^T$

3❧ Properties of the determinant

1 $\det(\mathbf{AB})=\det(\mathbf{A})\det(\mathbf{B})$

2 $\det(A^T)=\det(A)$

3 $\det(\mathbf{I}_n)=1$

and, if $\mathbf{A}$ is invertible,

4 $\det(\mathbf{A}^{-1})=\frac{1}{\det(\mathbf{A})}$

4❧ Nonlinear dynamical systems. The Jacobian:

$$\mathbf{M}=\begin{pmatrix}\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n}\\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n}\end{pmatrix}$$

10. Descriptive statistics

1☙ The average. For a sample of observations $x_1,\ldots,x_n$, the average is: $\overline{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$. If $f(x)$ is the relative frequency with which a value x occurs in the sample, then the average is also written $\overline{x}=\sum_{\text{All}\,x} f(x)x$. The two expressions will be identical if no binning is used to calculate the relative frequencies (e.g. as is the case in discrete variables).

2☙ One random variable

Variance	$v(x)=\frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})^2$
Standard deviation	$s(x)=\sqrt{v(x)}$
Skewness	$\frac{1}{ns^3}\sum_{i=1}^{n}(x_i-\overline{x})^3$
Kurtosis	$\frac{1}{ns^4}\sum_{i=1}^{n}(x_i-\overline{x})^4-3$

3☙ Two random variables

Covariance	$\operatorname{cov}(x,y)=\frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})$
Correlation	$r=\frac{\operatorname{cov}(x,y)}{s_x s_y}$

11. Sets and events

1☙ The empty set is written $\varnothing$ and the event space of an experiment is conventionally written Ω.

2☙ Comparisons between two sets

$A=B$	The set A is identical to the set B
$A\subset B$	The set A is a subset of the set B
$A\subseteq B$	Set A is a subset of, or the same as, set B
$A\not\subset B$	Set A is a not a subset of B

3✎ Set operators

$\bar{A}$ Negation: "not A"

$A \cup B$ Union of two sets: "A <u>or</u> B"

$A \cap B$ Intersection between two sets: "A <u>and</u> B"

4✎ Important relationships between events

$B = \bar{A}$: Complementary events

$A \cup B = \Omega$: Collectively exhaustive events

$A \cap B = \varnothing$: Mutually exclusive events

12. Rules of probability

1✎ For a set of mutually exclusive and collectively exhaustive events $E_1, E_2, E_3, \ldots, E_n$, it is always true that $\sum_{i=1}^{n} P(E_i) = 1$. If these events are equally likely, each with probability p, then $p = \frac{1}{n}$. Also, if two events are complementary, then $P(E) = 1 - P(\bar{E})$.

2✎ The probability of the union of two mutually exclusive events is $P(E_1 \cup E_2) = P(E_1) + P(E_2)$. The probability of the union of any two events is $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

3✎ Rules dealing with conditional probability

1. $P(E_2 \mid E_1) = \dfrac{P(E_1 \cap E_2)}{P(E_1)}$

2. $P(E_2 \mid E_1) = \dfrac{P(E_1 \mid E_2) P(E_2)}{P(E_1)}$ (Baye's law)

3. $P(E_1 \cap E_2) = P(E_1) P(E_2)$ (Independent events)

4. Total probability. Given the probabilities $P(E_1), \ldots, P(E_n)$ of n mutually exclusive and collectively exhaustive events, and the conditional probabilities $P(G \mid E_1), \ldots, P(G \mid E_n)$ referring to some other event G, then the total probability of G is $P(G) = \sum_{i=1}^{n} P(G \mid E_i) P(E_i)$

3✎ Bayesian probability.

$$P(H \mid data) = \frac{P(data \mid H) P(H)}{P(data)}$$

13. Probability distributions

1 Discrete random variables

Probability mass function (PMF): $f_X(x) = P(X = x)$

Properties of the PMF:

i) $0 \le f(x) \le 1 \ \ \forall x \in \Omega$ ii) $\sum_{All\ x \in \Omega} f(x) = 1$

Cumulative distr. function (CDF): $F_X(x) = P(X \le x)$

Properties of the CDF:

i) $\lim_{x \to -\infty} F(x) = 0$, ii) $\lim_{x \to \infty} F(x) = 1$,

iii) If $a \le b$, then $F(a) \le F(b)$

Relationship between the PMF and CDF:

$$f(x) = F(x) - F(x-1) \quad \& \quad F(x) = \sum_{i=-\infty}^{x} f(i)$$

2 Continuous random variables

Definition and properties of CDF: same as in discrete case.

Probability density function (PDF): defined as the derivative of the CDF. Properties of the PDF:

i) $f(x) \ge 0 \ \ \forall x \in \Omega$, ii) $P(a < X \le b) = \int_a^b f(x)dx$,

iii) $\int_{-\infty}^{\infty} f(x)dx = 1$

Relationship between PDF and CDF:

$$\frac{dF(x)}{dx} = f(x) \quad \& \quad F(x) = \int_{-\infty}^{x} f(s)ds$$

14. Expectation

1 Expectation of a random variable:

If X is discrete: $E(X) = \sum_{-\infty}^{\infty} xf(x)$

If X is continuous: $E(X) = \int_{-\infty}^{\infty} xf(x)dx$

2 Moments of a distribution:

If X is discrete: $E(X^m) = \sum_{-\infty}^{\infty} x^m f(x)$

If X is continuous: $E(X^m) = \int_{-\infty}^{\infty} x^m f(x)dx$

3 Expectation of a general function $g(X)$

If X is discrete: $E(g(X)) = \sum_{-\infty}^{\infty} g(x) f(x)$

If X is continuous: $E(g(X)) = \int_{-\infty}^{\infty} g(x) f(x) dx$

4 Relationship between moments and descriptive statistics

$\mu = E(X)$, $Var(X) = E(X^2) - E(X)^2$, $\mathrm{cov}(X,Y) = E\big((X-\mu_X)(Y-\mu_Y)\big)$

5 Manipulating expectations and variances

$E(X+Y) = E(X) + E(Y)$

$E(cX) = cE(X)$

$Var(cX) = a^2 Var(X)$

$E(XY) = E(X)E(Y)$ (independent variables)

$Var(X+Y) = Var(X) + Var(Y)$ (independent variables)

15. Combinatorics

1 The binomial coefficient counts he number of ways m of ordering x successes in a string of n trials: $m = \binom{n}{x} = \frac{n!}{x!(n-x)!}$

16. Discrete probability distributions

Name	*PMF*	*Mean*	*Variance*
Uniform $U(x_1, x_k)$	$\frac{1}{x_k - x_1 + 1}$	$\frac{x_1 + x_2}{2}$	$\frac{(x_k - x_1 + 1)^2 - 1}{12}$
Binomial $B(n,p)$	$\binom{n}{x} p^x q^{n-x}$	np	npq
Poisson Poisson(λ)	$\frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ
Geometric (x is no of trials) Geometric(p)	$f(x) = pq^{x-1}$	$1/p$	q/p^2
Geometric (x is no of failures) Geometric(p)	$f(x) = pq^{x-1}$	q/p	q/p^2
Negative binomial if x is no of trials NegB(k,p)	$f\binom{x-1}{k-1} p^k q^{x-k}$	k/p	kq/p^2
Negative binomial if x is no of failures	$\binom{k+x-1}{k-1} p^k q^x$	kp/q	kp/q^2

$X \sim \text{NegB}(k,p)$			
Multinomial Multinom$(n,\mathbf{p})$	$\frac{n!}{x_1!\cdots x_k!}p_1^{x_1}\cdots p_k^{x_k}$	$\mu_i = np_i$	$\sigma_i^2 = np_i q_i$.

17. Continuous probability distributions

Name	*PMF*	*Mean*	*Variance*
Uniform $U(a,b)$	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential $M(\lambda)$	$\lambda e^{-\lambda x}$	$1/\lambda$	$1/\lambda^2$
Gamma $Gamma(k,\lambda)$	$x^{k-1}\lambda^k \frac{e^{-\lambda x}}{\Gamma(k)}$	k/λ	k/λ^2
Beta $Beta(\alpha,\beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Normal $N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\frac{(\mu-x)^2}{\sigma^2}}$	μ	σ^2
t-distribution $t(n)$	$\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$	0	$\begin{cases} \infty & \text{if } 1<n\le 2 \\ \frac{n}{n-2} & \text{if } n>2 \end{cases}$
Lognormal $LogN(\mu,\sigma^2)$	$\frac{1}{y\sqrt{2\pi\sigma^2}}e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}$	$e^{\mu+\frac{1}{2}\sigma^2}$	$(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$
Chi-square $\chi^2(n)$	$\frac{x^{n/2-1}}{2^{n/2}}\frac{e^{-x/2}}{\Gamma(n/2)}$	n	$2n$